from (13), with an additional equation arising from (13) for $j = 0$ when u_{-1} is obtained from (14). The remaining equation is that for $\dot{z}$, Eq. (9).

Most computing machines are equipped with powerful subprograms for solving systems of ordinary differential equations. These programs are usually designed to select automatically the interval of integration based on criteria designed to bound the truncation error. Since the problems under consideration are often characterized by rapidly decaying transients, it follows that these problems allow vastly differing integration intervals to maintain a fixed truncation error. Explicit finite-difference methods require adjusting the space increment when varying the time increment to maintain a fixed stability ratio. For this reason, and because of a lack of criteria for adjusting step size in both explicit and implicit difference schemes, the method of lines seems to be the simplest, most efficient, and most flexible method for the type of problem presented herein. The greatest advantage, however, is its ability to find the position of the moving boundary without iteration. Variations of the procedure described in this paper have been used successfully by the author on problems involving the recession of a burning fluid and the solidification of explosives. In addition, the method can easily be extended to heat flow problems involving composite materials or to materials having thermal properties dependent on temperature.

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Reply by Author to H. J. Breaux

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In his comment[1] on Ref. 2, Breaux stated that the method presented was restricted by several conditions. The method in itself is not restricted, as claimed by Breaux; only the presentation in the Note was restricted for purposes of brevity.

A footnote[2] in my Note stated that the method has been extended to encompass radial heat flow in cylinders and spheres, finite-difference methods for inward and outward surface recession, equations describing cylindrical and spherical sublimation-conduction for several typical composite material arrangements, equations defining criteria for stopping surface recession, and equations to determine heat flow after recession terminates. These techniques all have been used successfully in the past and no shortcomings nor difficulties have been encountered.

As stated in Ref. 2, the equations presented are applicable only to the first interior nodal point where this point is located less than one Δ space increment from the receding surface. The surface temperature is constant (variable with time if one desires) and the other nodal points, with the exception of the second nodal point, are calculated using general explicit finite-difference equations. Once the recession front reaches the original first interior nodal point (T_2), then the equations presented[2] are applied to the original third nodal point (T_3), as stated on p. 1680[2] after Eq. (3). This procedure of handling the first interior nodal point temperature is continued until the receding surface is within one Δ increment of the back surface or an interface as stated in the Note[2] on p. 1680. Special equations, of no greater complexity, must then be applied.

The reason that the presentation was restricted to cases where the temperature of the receding surface remained constant during the recession process and the material thermal properties did not vary with temperature was to allow for comparisons of data to exact solution results. Variable thermal property data have been used and the receding surface temperature varying with time can be very easily incorporated.

To circumvent compressing the grid and encountering small time increments, especially as the receding surface approaches a backside or substructure interface, the nonshifting grid technique was considered advantageous. Inherent to a nonshifting grid is computational efficiency and a fixed stability ratio. If the computed temperatures are to be compared with empirical thermocouple data from a receding material, the fixed grid is most desirable and simple to arrange.

As stated in the Note,[2] the procedure requires a negligible increase in computer time over an identical nonrecession case, primarily because all calculations internal to the first interior node are made by ordinary explicit finite differences. The computer program efficiency is thus comparable to a nonrecession explicit finite difference program. The efficiency has been verified by past experience in using the program.

This recession-conduction procedure is extremely simple for extending general explicit finite-difference heat conduction routines to provide analytical capability in calculating the temperature of a structure during surface recession.

References

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Comment on "Inner Region of Transpired Turbulent Boundary Layers"

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In a recent Note Stevenson[1] discusses the disparity between the theoretical friction factors for the transpired turbulent boundary layer predicted in Ref. 2 and those in Ref. 3. The

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